

PNPI-TH-2411/01
 ITEP-TH-11/01
 TPI-MINN 01/14
 hep-th/0103222

Confinement Near Argyres-Douglas Point in $\mathcal{N}=2$ QCD and Low Energy Version of AdS/CFT Correspondence

Alexei Yung

*Petersburg Nuclear Physics Institute, Gatchina, St. Petersburg 188300;
 Institute of Experimental and Theoretical Physics, Moscow, 117259;
 Theoretical Physics Institute, University of Minnesota, Minneapolis, MN
 55455*

Abstract

We study Abrikosov-Nielsen-Olesen (ANO) flux tubes on the Higgs branch of $\mathcal{N}=2$ QCD with $SU(2)$ gauge group and $N_f = 2$ flavors of fundamental matter. In particular, we consider this theory near Argyres-Douglas (AD) point where the mass of monopoles connected by these ANO strings become small. In this regime the effective QED which describes the theory on the Higgs branch becomes strongly coupled. We argue that the appropriate description of the theory is in terms of long and thin flux tubes (strings) with small tension. We interpret this as another example of duality between field theory in strong coupling and string theory in weak coupling. Then we consider the non-critical string theory for these flux tubes which includes fifth (Liouville) dimension. We identify CFT at the AD point as UV fix point corresponding to AdS metric on the 5d “gravity” side. The perturbation associated with the monopole mass term creates a kink separating UV and IR behavior. We estimate the renormalized string tension and show that it is determined by the small monopole mass. In particular, it goes to zero at the AD point.

1 Introduction

One of the most important physical outcomes of the Seiberg-Witten theory [1, 2] is the demonstration of confinement of electric charges via monopole condensation. Although the idea of confinement as a dual Meissner effect was suggested many years ago by Mandelstam and 't Hooft [3] only relatively recent progress in the understanding of the electromagnetic duality in $\mathcal{N}=2$ supersymmetry allowed Seiberg and Witten to present the quantitative description of this phenomenon [1, 2] .

Let us recall the basic idea of Mandelstam and 't Hooft [3]. Once monopoles (charges) condense the electric (magnetic) flux is confined in the Abrikosov-Nielsen-Olesen (ANO) flux tube [4] connecting heavy trial electric (magnetic) charge and anti-charge. The energy of the ANO string increases with its length. This ensures increasing confinement potential between heavy electric (magnetic) charge and anti-charge.

In the Seiberg-Witten theory this confinement scenario is realized in two possible ways. First, in the pure gauge theory near the monopole (dyon) singularity upon breaking $\mathcal{N}=2$ supersymmetry by the small mass term of the adjoint matter [1]. In this case monopoles condense and electric charges are confined by electric flux tubes.

Second, in the unbroken $\mathcal{N}=2$ theory with N_f flavors of the fundamental matter with degenerative masses on Higgs branches [2]. In this case, say at large masses of the fundamental matter electric charges condense on the Higgs branch while magnetically charged dyons confined by magnetic flux tubes [5, 6].

Given this progress it is still unclear if the confinement in the Seiberg-Witten theory can be taken as a model (at least qualitatively) for the confinement in QCD. The point is that the confinement in the Seiberg-Witten theory has several unwanted properties we do not expect to have in QCD (see [7] for a recent review).

In particular, one group of such properties of confinement in Seiberg-Witten theory is related to the large value of the ANO string tension T . In terms of the photon mass m_γ of the effective low energy $\mathcal{N}=2$ QED it is given by

$$T \sim \frac{m_\gamma^2}{g^2}. \quad (1.1)$$

At small values of QED coupling g^2 it is much larger than m_γ . As a conse-

quence the low energy hadron spectrum, say near the monopole point consists of relatively light photon and monopoles (which we would like to interpret as glueballs) of mass m_γ and heavy hadrons built of quarks connected by ANO strings with mass of order of \sqrt{T} [7]¹. In contrast, in QCD we have light $\bar{q}q$ states while the candidates for glueballs are relatively heavier.

Another problem related to the large value of ANO string tension (1.1) is the non-linear behavior of Regge trajectories. The transverse size of a string is of order of $1/m_\gamma$. This is much larger than its length which is of order of $1/\sqrt{T}$ for small hadron spins j . Therefore, the string is not developed (grows thick and short) and the $\bar{q}q$ state looks more like spherically symmetric soliton rather than a string. This is the reason for the non-linear behavior of Regge trajectories in wide region of spins $j \lesssim 1/g^2$ [7].

The purpose of this paper is to overcome the above mentioned problems related to the large value of the string tension (1.1). We suggest a regime in the Seiberg-Witten theory in which ANO strings become light (almost tensionless).

To be more specific, we consider the second of the above mentioned scenarios of confinement which arises on the Higgs branch. Namely, we consider $\mathcal{N}=2$ gauge theory with $SU(2)$ gauge group and two hypermultiplets of the fundamental matter (we call them quarks). If the masses of quarks are equal the Higgs branch is developed. It touches the Coulomb branch at the singular point where some quarks become massless [2] (we give a brief review of Higgs branches in the next section). The effective low energy description of the theory near the root of Higgs branch is given by $\mathcal{N}=2$ QED. When scalar quarks develop vev's on the Higgs branch the effective QED is in the Higgs phase. The formation of ANO flux tubes in this vacuum leads to confinement of monopoles (any dyons with non-zero magnetic charge) [5, 6]²

At large values of bare quark mass m monopoles are heavy and can be viewed as heavy trial particles to probe confinement. It is a challenging problem to see what happens to confinement if we reduce the monopole mass and make monopoles dynamical. If ANO flux tubes still exist in this

¹This is true if we take the bare mass of quarks to be small, the possibility discussed in detail below.

²Strictly speaking this terminology refers to large values of bare quark masses. For small quark masses (below Argyres-Douglas point) quantum numbers of particles are changed because of monodromies [1] and now monopoles are condensed at the Higgs branch while electric charges are confined [8]. To avoid confusion in this paper we use the terminology which refers to large quark masses.

regime and have finite transverse size we would still expect confinement of monopoles to occur, although the Wilson loop does not show the area law any longer (flux tubes can be broken by light monopole-anti-monopole pairs). In this setup the problem of confinement becomes similar to that in QCD.

In this paper we study what happens to ANO flux tubes on the Higgs branch of the Seiberg-Witten theory once we reduce m and eventually come close to the Argyres-Douglas (AD) point. These points were originally introduced in the moduli/parameter space of $\mathcal{N}=2$ theories as points where two singularities with mutually non-local light states on the Coulomb branch coalesce [9, 10, 11]. It is believed that the theory in the AD point flows in the infrared to a nontrivial superconformal theory (let us call it CFT_{AD}). We consider the theory on the Higgs branch where massless scalar quarks develop vev v . AD point corresponds to the value of the bare quark mass m equal to some critical value m_{AD} at which monopoles also become massless.

When we come close to the AD point our effective QED description is no longer valid because QED enters a strong coupling regime. Our main proposal in this paper is that the ANO string tension T becomes small, $T \ll v^2$. We give arguments based on what we know about the effective sigma model on the Higgs branch in favor of this conjecture. As we explained above the small value of string tension eliminates certain important “disadvantages” of confinement in the Seiberg-Witten theory making it similar to the one we expect in QCD.

Once \sqrt{T} is much smaller than the inverse transverse size of the flux tube the correct low energy description of the theory is in terms of long and thin strings. We apply methods of non-critical string theory developed mostly by Polyakov [12, 13] to our ANO flux tube. We consider this as another example of duality between field theory (QED, in the case at hand) at strong coupling and string theory at weak coupling.

The non-critical string theory contains curved fifth (Liouville) coordinate u [12, 13] which is associated with RG scale in field theory [14]. We suggest a low energy version of AdS/CFT correspondence [14, 15, 16] in which the AdS metric at large u on the 5d “gravity” side corresponds to CFT_{AD} on the field theory side in the UV limit. Note, that we use the word gravity in quotation marks here because it has nothing to do with the real gravity at the Planck scale. In this paper we discuss “gravity inside hadrons” [13] which appears at the hadron scale $\sim \sqrt{T}$.

Then we consider breaking of the conformal invariance induced by small monopole mass near the AD point (it is determined by $m - m_{AD}$). On the

“gravity” side we associate this perturbation with a scalar field in 5d “gravity”. We study 5d “gravity” equations of motion with this scalar included and find the scale u_{CB} at which the kink separating the UV and IR behavior destroys the AdS metric. Using u_{CB} we estimate the renormalized ANO string tension. It turns out to be small, proportional to the monopole mass. This result shows selfconsistency of our initial conjecture. In particular, T goes to zero at the AD point.

Then we verify that the radius of AdS space is large in string units justifying the “gravity” approximation.³ We also make two estimates of the string coupling constant g_s , one from the “gravity” side and another one from the field theory side. Although our accuracy is not enough to show quantitative agreement both calculations shows that g_s is small, $g_s \ll 1$.

The paper is organized as follows. In sect. 2 we review quasiclassical results on ANO flux tubes on Higgs branches of Seiberg-Witten theory. In sect. 3 we discuss field/string theory duality near the AD point and introduce 5d “gravity” description. In sect. 4 we consider perturbation of AdS_5 metric and estimate the renormalized string tension. Sect. 5 contains our conclusions and discussion.

2 ANO strings on Higgs branch in the quasiclassical regime

In this section we review quasiclassical results obtained for ANO flux tubes on Higgs branch of $\mathcal{N}=2$ QCD with gauge group $SU(2)$ and $N_f = 2$ flavors of fundamental matter (quarks) with common bare mass parameter m [6]. First, we briefly review the effective theory on the Higgs branch [2].

2.1 Higgs branch

The $\mathcal{N}=2$ vector multiplet of the theory at hand on the component level consists of the gauge field A_μ^a , two Weyl fermions $\lambda_1^{\alpha a}$ and $\lambda_2^{\alpha a}$ ($\alpha = 1, 2$) and the complex scalar φ^a , where $a = 1, 2, 3$ is the color index. Fermions form a doublet $\lambda_f^{\alpha a}$ with respect to global $SU(2)_R$ group, $f = 1, 2$.

The scalar potential of this theory has a flat direction. The adjoint scalar field develop an arbitrary vev along this direction breaking $SU(2)$ gauge

³Note, we do not use the logic of large N [14] in this paper.

group down to $U(1)$. We choose $\langle \varphi^a \rangle = \delta^{a3} \langle a \rangle$. The complex parameter $\langle a \rangle$ parameterize the Coulomb branch. The low energy effective theory generically contains only the photon $A_\mu = A_\mu^3$ and its superpartners: two Weyl fermions $\lambda_f^{3\alpha}$ and the complex scalar a . This is massless short vector $\mathcal{N}=2$ multiplet. It contains 4 boson + 4 fermion states. W-boson and its superpartners are massive with masses of order of $\langle a \rangle$.

Quark hypermultiplets has the following structure. They consist of complex scalars $q^{k f A}$ and fermions $\psi^{k \alpha A}$, $\tilde{\psi}_{A k}^{\dot{\alpha}}$, where $k = 1, 2$ is the color index and $A = 1, \dots, N_f$ is the flavor one. Scalars form a doublet with respect to $SU(2)_R$ group. All these states are in the BPS short representations of $\mathcal{N}=2$ algebra on the Coulomb branch with $4 \times N_c \times N_f = 16$ real boson states (+ 16 fermion states).

Coulomb branch has three singular points where monopoles, dyons or charges become massless. Two of them correspond to monopole and dyon singularities of the pure gauge theory. Their positions on the Coulomb branch are given by [2]

$$u_{m,d} = \pm 2m\Lambda_2 - \frac{1}{2}\Lambda_2^2, \quad (2.1)$$

where $u = \frac{1}{2}\langle \varphi^{a^2} \rangle$ and Λ_2 is the scale of the theory with $N_f = 2$. In the large m limit $u_{m,d}$ are approximately given by their values in the pure gauge theory $u_{m,d} \simeq \pm 2m\Lambda_2 = \pm 2\Lambda^2$, where Λ is the scale of $N_f = 0$ theory.

The charge singularity corresponds to the point where half of quark states becomes massless. We denote them q^{fA} and $\psi^{\alpha A}$, $\tilde{\psi}_A^{\dot{\alpha}}$ dropping the color index. They form $N_f = 2$ short hypermultiplets with $4 \times N_f = 8$ real boson states. The rest of quark states acquire large mass $2m$ and we ignore them in the low energy description. The charge singularity appears at the point

$$a = -\sqrt{2} m \quad (2.2)$$

on the Coulomb branch. In terms of variable u (2.2) reads

$$u_c = m^2 + \frac{1}{2}\Lambda_2^2. \quad (2.3)$$

Strictly speaking, we have $2 + N_f = 4$ singularities on the Coulomb branch. However, two of them coincides for the case of two flavors of matter with the same mass.

The effective theory on the Coulomb branch near the charge singularity (2.2) is given by $\mathcal{N} = 2$ QED with light matter fields q^{fA} , and their superpartners as well as the photon multiplet.

The charge singularity (2.2) is the root of the Higgs branch [2]. To find it we impose D -term and F -term conditions which look like

$$\bar{q}_{Ap}(\tau^m)_f^p q^{fA} = 0, \quad m = 1, 2, 3. \quad (2.4)$$

(Here m is an adjoint $SU(2)_R$ index, not to be confused with color indices.) This equation determines the Higgs branch (manifold with $\langle q \rangle \neq 0$) which touches the Coulomb branch at the point (2.2). It has non-trivial solutions for $N_f \geq 2$ [2]. This is the reason why we choose $N_f = 2$ for our discussion.

The low energy theory for boson fields near the root of the Higgs branch is given by

$$S_{\text{boson}}^{\text{root}} = \int d^4x \left\{ \frac{1}{4g^2} F_{\mu\nu}^2 + \bar{\nabla}_\mu \bar{q}_{Af} \nabla_\mu q^{fA} + \frac{g^2}{8} [\text{Tr} (\bar{q} \tau^m q)]^2 \right\}, \quad (2.5)$$

where trace is calculated over flavor and $SU(2)_R$ indices. Here $\nabla_\mu = \partial_\mu - in_e A_\mu$, $\bar{\nabla}_\mu = \partial_\mu + in_e A_\mu$, the electric charge $n_e = 1/2$ for fundamental matter fields.

This is an Abelian Higgs model with last interaction term coming from the elimination of D and F terms. The QED coupling constant g^2 is small near the root of the Higgs branch if m is not close to its AD value m_{AD} . The effective theory (2.5) is correct on the Coulomb branch near the root of the Higgs branch (2.2) or on the Higgs branch not far away from the origin at $\langle q \rangle = 0$.

Once $|\langle q \rangle|^2 = v^2 \neq 0$ on the Higgs branch the $U(1)$ gauge group in (2.5) is broken and the photon acquires the mass

$$m_\gamma^2 = \frac{1}{2} g^2 v^2 \quad (2.6)$$

It is clear that the last term in (2.5) is zero on fields q which satisfy constraint (2.4). This means that moduli fields which develop vev's on the Higgs branch are massless, as expected. It turns out that there are four real moduli fields q (out of 8) which satisfy the constraint (2.4) [2]. They correspond to the lowest components of one short hypermultiplet.

The other quark fields (4 real boson states + fermions) acquire the mass of the photon (2.6). Together with states from the photon multiplet they form one long (non-BPS) $\mathcal{N}=2$ multiplet (cf. [17]). It has 8 boson + 8 fermion states. The reason why the long multiplet appears is easy to understand. Electric charge is screened by the quark condensation on the Higgs branch.

Therefore, the central charges of $\mathcal{N}=2$ algebra are zero now. That means that there is no BPS particles any longer. Note, that the multiplet of moduli fields is short (4 boson + 4 fermion states) because it is massless.

We can parameterize massless moduli fields as

$$q^{f\dot{A}}(x) = \frac{1}{\sqrt{2}} \sigma_{\alpha}^{f\dot{A}} \phi_{\alpha}(x) e^{i\alpha(x)}. \quad (2.7)$$

Here $\phi_{\alpha}(x)$, $\alpha = 1 \dots 4$ are four real moduli fields. It is clear that fields (2.7) solve (2.4). The common phase $\alpha(x)$ in (2.7) is the U(1) gauge phase. Once $\langle \phi_{\alpha} \rangle = v_{\alpha} \neq 0$ on the Higgs branch the U(1) group is broken and $\alpha(x)$ is eaten by the Higgs mechanism. Say, in the unitary gauge $\alpha(x) = 0$. In the next subsection we consider vortex solution for the model (2.5). Then $\alpha(x)$ is determined by the behavior of the gauge field at the infinity.

Once $v_{\alpha} \neq 0$ we expect monopoles (they are heavy at $m \gg \Lambda_2$) to confine via formation of vortices which carry the magnetic flux. The peculiar feature of the theory (2.5) is the absence of the Higgs potential for the moduli fields ϕ_{α} . Therefore, the Higgs phase of the theory in (2.5) is the limiting case of type I superconductor with the vanishing Higgs mass. In the next subsection we will review the peculiar features of ANO vortices in this model.

If we consider the low energy limit of the theory at energies much less than the photon mass (2.6) we can integrate out massive fields. Then the effective theory is a σ -model for massless fields ϕ_{α} which belong to 4-dimensional hyper-Kähler manifold R^4/Z_2 . The metric of this σ -model is flat [2, 18], there are, however, higher derivative corrections induced by instantons [19, 20].

2.2 ANO string

Now let us review classical solution for ANO vortices in the model (2.5) [6]. Without loss of generality we take vev's of ϕ_{α} $v_{\alpha} = (v, 0, 0, 0)$. Moreover, we drop fields ϕ_2, ϕ_3 and ϕ_4 together with massive scalars from (2.5) because they are irrelevant for the purpose of finding classical vortex solutions. Thus, we arrive at the standard Abelian Higgs model

$$S_{AH} = \int d^4x \left\{ \frac{1}{4g^2} F_{\mu\nu}^2 + |\nabla_{\mu} q|^2 + \lambda(|q|^2 - v^2)^2 \right\}, \quad (2.8)$$

for the single complex field q with quartic coupling $\lambda = 0$. Here

$$q(x) = \phi_1(x) e^{i\alpha(x)}. \quad (2.9)$$

Following [6] consider first the model (2.8) with small λ , so that the Higgs mass $m_H \ll m_\gamma$ ($m_H^2 = 4\lambda v^2$). Then we take the limit $m_H \rightarrow 0$.

To the leading order in $\log m_\gamma/m_H$ the vortex solution has the following structure in the plane orthogonal to the string axis [6]. The electromagnetic field is confined in a core with the radius

$$R_g^2 \sim \frac{1}{m_\gamma^2} \ln^2 \frac{m_\gamma}{m_H} . \quad (2.10)$$

The scalar field is close to zero inside the core. Instead, outside the core, the electromagnetic field is vanishingly small while the scalar field slowly (logarithmically) approaches its boundary value v . The result for the string tension is [6]

$$T_\lambda = \frac{2\pi v^2}{\ln m_\gamma/m_H} . \quad (2.11)$$

The main contribution to the tension in (2.11) comes from the logarithmic “tail” of the scalar field.

The results in (2.10), (2.11) mean that if we naively take the limit $m_H \rightarrow 0$ the string becomes infinitely thick and its tension goes to zero [6]. This means that there are no strings in the limit $m_H = 0$. The absence of ANO strings in theories with flat Higgs potential was first noticed in [21, 22].

One might think that the absence of ANO strings means that there is no confinement on Higgs branches. As we will see now this is not the case [6]. So far we have considered infinitely long ANO strings. However the setup for the confinement problem is slightly different [6]. We have to consider monopole–anti-monopole pair at large but finite separation L . Our aim is to take the limit $m_H \rightarrow 0$. To do so let us consider ANO string of the finite length L within the region

$$\frac{1}{m_\gamma} \ll L \ll \frac{1}{m_H} . \quad (2.12)$$

Then it turns out that $1/L$ plays the role of the IR -cutoff in Eqs. (2.10) and (2.11) instead of m_H [6]. Now we can safely put $m_H = 0$.

The result for the electromagnetic core of the vortex becomes

$$R_g^2 \sim \frac{1}{m_\gamma^2} \ln^2 m_\gamma L , \quad (2.13)$$

while its string tension is given by [6]

$$T_0 = \frac{2\pi v^2}{\ln m_\gamma L} . \quad (2.14)$$

We see that the ANO string becomes "thick" but still its transverse size R_g is much less than its length L , $R_g \ll L$. As a result the potential between heavy well separated monopole and anti-monopole is still confining but is no longer linear in L . It behaves as [6]

$$V(L) = 2\pi v^2 \frac{L}{\ln m_\gamma L} . \quad (2.15)$$

As soon as the potential $V(L)$ is an order parameter which distinguishes different phases of a theory (see, for example, review [23]) we conclude that we have a new confining phase on the Higgs branch of the Seiberg–Witten theory. It is clear that this phase can arise only in supersymmetric theories because we do not have Higgs branches without supersymmetry.

These quasiclassical results are valid if the effective QED coupling is small, $g^2 \ll 1$. It is small if two conditions are satisfied. First, $v \ll \Lambda_2$ ensures that W-bosons are massive and we can ignore them and use the effective QED description. Second, is that m is not close to m_{AD} , which ensures that monopoles/dyons are massive. In the next section we relax the second condition and study what happen to ANO strings if we go close to AD point.

Now let us comment on why we choose this relatively complicated type of ANO string for our study of what happens to confinement near AD point. The reason is that $\mathcal{N}=2$ supersymmetry is unbroken on the Higgs branch. We will use this heavily in the next section. Alternatively, one could consider the confinement scenario near the monopole point which arises upon breaking $\mathcal{N}=2$ supersymmetry down to $\mathcal{N}=1$ by the mass term of the adjoint matter. In this case ANO strings appear to be "almost" BPS saturated once the breaking is small [5, 17]. However, if we reduce quark mass going to AD point the monopole condensate vanishes showing deconfinement [24]. We can try to find a regime in which monopole condensate is fixed while quark mass goes to zero. However, it is easy to show that in this regime $\mathcal{N}=2$ breaking becomes strong [7].

3 ANO string at the AD point

In this section we discuss what happens to ANO strings if we go close to AD point. On the Coulomb branch the mass of the monopole is given by the BPS formula [1] $m_m = \sqrt{2}|a_D|$, where a_D is the dual variable to a . Now let us take m close to m_{AD} so the monopole singularity collides with the root of the Higgs branch (charge singularity). From (2.1) and (2.3) we learn that

$$m_{AD} = \Lambda_2. \quad (3.1)$$

Since $a_D = 0$ in the monopole point, while charges are massless in the charge singularity we have simultaneously both charges and monopoles becoming massless at $m = \Lambda_2$. The theory at AD point on the Coulomb branch flows in the IR to a non-trivial interacting fixed point [9, 10] which we call CFT_{AD}^C . The superscript C indicates here that we are talking about the CFT on the Coulomb branch. The conformal dimension of a_D equals to 1 near AD point, while the conformal dimension of $(m - \Lambda_2)$ is $2/3$ [10]. Thus, we conclude that the monopole mass behaves as

$$m_m \sim \frac{(m - \Lambda_2)^{3/2}}{\Lambda_2^{1/2}} \quad (3.2)$$

when $m \rightarrow \Lambda_2$.

Let us go to the Higgs branch taking $v \neq 0$. Now we have two scales in the problem v and m_m determined by $m - \Lambda_2$ via (3.2). If $m_m \gg v$ then monopoles are heavy. The monopole multiplet contains 4 boson + 4 fermion states. Still monopoles cannot be BPS saturated on the Higgs branch because central charges of $\mathcal{N}=2$ algebra are zero. They are confined by ANO flux tubes and instead of monopoles we see hadrons built of open ANO string states with monopoles and anti-monopoles attached to string ends. These are, of course, non-BPS states. Note, that as we discussed in the Introduction, in fact, strings are not developed in this regime. They grow short and thick provided the effective QED coupling is small, see (2.13), (2.14).

More specifically, if $m_m \gg v$ the effective coupling is of order of

$$g^2 \sim -\frac{1}{\log(m_\gamma/\Lambda_2)}, \quad (3.3)$$

frozen at the photon mass scale. Thus, $g^2 \ll 1$ at $v \ll \Lambda_2$. Instead, near the monopole point on the Coulomb branch the dual coupling constant is small,

thus g^2 is large

$$g^2 \sim -\log(m_m/\Lambda_2). \quad (3.4)$$

It is clear from last two equations that the QED coupling constant on the Higgs branch increases as we reduce $m - \Lambda_2$ and eventually we enter the strong coupling regime, $g^2 \sim 1$ at $m_m \sim v$.

3.1 Tensionless strings and QED/string theory duality

Now let us discuss what happens if we reduce m_m well below v making monopole much lighter than photon. At first glance, the natural guess is that ANO string tension stays large,

$$T \sim \frac{v^2}{\log vL} \quad (3.5)$$

as it is suggested by the quasiclassical result (2.14). This seems natural because the string is “built” of quarks and electromagnetic field and seems to have nothing to do with monopoles. However, as we will now show the guess in (3.5) is not correct.

Suppose we keep $m_m \ll v$ but do not go directly to AD point and integrate out hadrons built of monopoles together with photon multiplet. Then we are left with a sigma model for massless quark moduli. As soon as the Higgs branch is a hyper-Kähler manifold its metric is determined uniquely and cannot receive corrections. In fact, it is known to be flat [2, 18]. However, there are higher derivative corrections. It is clear that higher derivative corrections encode all the information about massive states we have integrated out.

On dimensional grounds higher derivative corrections can go in powers of

$$\frac{\partial^2}{v^2} \quad (3.6)$$

or in powers of

$$\frac{\partial^2}{(m - \Lambda_2)^2}. \quad (3.7)$$

The difference $(m - \Lambda_2)$ appears here because it is clear that the theory can have singularities only at the AD value of m , $m = \Lambda_2$. At any other values of m the theory is smooth.

Higher derivative corrections of type (3.6) become singular at $v = 0$ (on the Coulomb branch) showing that certain states become massless in this limit. Photon multiplet is an example of such a state. Higher derivative corrections of type (3.7), if present, would signal that some states become massless at AD point on the Higgs branch at non-zero v . If (3.5) were correct there would be no such corrections because all hadrons built of monopoles are heavy with masses of order of $\sqrt{T} \sim v$ and cannot produce singularities of type (3.7).

Now the question is whether there are higher derivative corrections of type (3.7). This problem was studied in [19]. In particular, in [19] higher derivative corrections on the Higgs branch induced by one instanton were calculated. Consider large m , $m \gg \Lambda_2$ far away from the AD point. Then the Higgs branch is in the weak coupling and quasi-classical methods can be applied. The holomorphic one instanton induced corrections appear to be non-zero and proportional to powers of ∂/m and Λ_2/m . Note, that v is considered large in [19], $v \gg m$. Thus, these instanton corrections are really of type (3.7) rather than of type (3.6). Now if we reduce m going to AD point these corrections blow up showing a singularity at $m = \Lambda_2$. This singularity should correspond to some extra states (besides quark moduli) becoming massless. These extra states cannot be just monopoles because monopoles are in the confinement phase. They are bound into hadrons by ANO strings.

The plausible suggestion is that some hadrons built of monopoles (ANO string states) become massless. This could happen only if the ANO string becomes tensionless at the AD point. Thus, we conclude that (3.5) is not correct and suggest instead that

$$T \ll v^2 \tag{3.8}$$

at $m_m \ll v$. In particular, we need

$$T(m_m \rightarrow 0) \rightarrow 0. \tag{3.9}$$

Note, that alternatively, we could suggest that the string tension stays large but some of string states become massless. Definitely this would be the case for a critical string. However, for a non-critical string (moreover, for a string without world sheet conformal invariance) this hardly can happen.

In particular, (3.9) means that strictly at the AD point the theory flows in the IR to a non-trivial conformal field theory of interacting massless

quark moduli with massless string states. We call this theory CFT_{AD}^H where H stands for the Higgs branch. This is a descendant of CFT_{AD}^C which is a CFT at the AD point on the Coulomb branch.

The conclusion in eqs. (3.8), (3.9) is quite a dramatic one. It means that we loose confinement as we reduce mass of confining matter. In particular, (3.9) means that size of hadrons built of monopoles becomes infinite as we approach AD point. This is in a contradiction with the standard point of view that, say, confinement in QCD is insensitive to the quark mass. We assume the proposal (3.8), (3.9) below in this paper and check its selfconsistency.

The proposed behavior although surprising is quite similar to what happens to monopoles in the Seiberg-Witten theory. At certain point on the Coulomb branch the monopole becomes massless. Its size stays small, of order of the inverse W-boson mass so we can consider monopole as a point-like particle to be included in the effective low energy theory. Similar to that, in the case at hand, at certain point of the parameter space ($m = \Lambda_2$) the ANO string becomes tensionless, while its transverse size R_g remains small, determined by the mass of the photon ($\sim v^{-1}$).

This means that ANO strings are now long and thin (with typical length $L \sim 1/\sqrt{T}$). This is exactly what is assumed for a string in the string theory. Thus, we expect that appropriate description of our theory is in terms of non-critical theory of ANO strings. We interpret this as a duality between field theory ($\mathcal{N}=2$ QED) at strong coupling and the string theory at weak coupling ⁴.

In the conclusion of this subsection let us compare the physics on the Higgs branch at the AD point with the one we have at the AD point of $N_f = 1$ theory with $\mathcal{N}=2$ supersymmetry broken down to $\mathcal{N}=1$ by the mass term of the adjoint matter [24]. Suppose we are at the monopole vacuum turning the quark mass parameter in a way to ensure the collision of the monopole vacuum with the charge one. Then the monopole condensate goes to zero in the AD point [24]. The monopole condensate sets the mass scale of all light states in the theory, thus, all of them become massless (including photon). In particular, the ANO string becomes tensionless, however its transverse size (given by the inverse photon mass) goes to infinity. Therefore, it cannot be considered as a stringy object, in fact, it disappears [24].

On the Higgs branch in the theory at hand the physics is quite different. At the AD point ANO strings becomes tensionless too, but their transverse

⁴We confirm in the next section that the string coupling g_s appears to be small.

size remains small, of order of v^{-1} . The string remains a localized object in transverse directions. In fact, the mass of the photon ($\sim v$) plays now the role of the UV cutoff for our low energy effective string theory. Below we describe this string theory and use it to calculate the string tension T .

3.2 5d string theory and AdS metric

One can develop a string theory representation for the ANO string in the quasiclassical regime $g^2 \ll 1$. This is done in [25, 26] and [6] for cases of strings in the type II superconductor, BPS-strings and strings in the type I superconductor respectively. The common feature of these representations is that the leading term of the world sheet action is the Nambu–Goto term

$$S_{\text{str}} = T \int d^2\sigma \left\{ \sqrt{g^{\text{ind}}} + \text{higher derivatives} \right\}, \quad (3.10)$$

where $g_{ij}^{\text{ind}} = \partial_i x_\mu \partial_j x_\mu$ is the induced metric ($i, j = 1, 2$). Higher derivative corrections in (3.10) include the Jacobian term [27, 28], rigidity term [29] etc. These terms contain powers of ∂/m_γ . For thin strings m_γ is large and these corrections can be neglected in the action (3.10). However, as we explained in the Introduction for the ANO vortex in the semiclassical regime $\partial^2/m_\gamma^2 \sim T/m_\gamma^2 \sim 1/g^2 \gg 1$ (see (1.1)). Hence, higher derivative corrections blow up in (3.10) and the string approximation is not acceptable. In particular, there is no world sheet conformal invariance.

The QED coupling is large at the AD point so there is no hope to use quasiclassical analysis to derive the string theory for ANO flux tubes from QED. Therefore, we take another route. We have to construct this string theory imposing the world sheet conformal invariance. The latter requirement follows from (3.8). To see this note, that if there were no world sheet conformal invariance, the string tension would get renormalized [29] and become of order of the string theory UV cutoff, which is v^2 . This is in a contradiction with (3.8).

To maintain the world sheet conformal invariance is quite a problem for the string moving in the space with a non-critical dimensionality. The Liouville coordinate does not decouple and a string should be considered as moving in the 5 dimensional space [31]. Even this “high price” appeared to be not enough. For $d \geq 1$ the string turns out to be unstable.

The way out was found by Polyakov who conjectured that the fifth coordinate should be curved [12, 13]. The bosonic part of the string action looks

like [12, 13]

$$S_{\text{str}} = T_0 \int d^2\sigma \left[a(y)(\partial_i x^\mu)^2 + (\partial_i y)^2 + \Phi(y)R_2 + \dots \right]. \quad (3.11)$$

Here y is the Liouville coordinate, R_2 is the world sheet curvature and Φ is the dilaton depending on the fifth coordinate only. Dots stands for other possible background fields. Of course, the metric in the 4-dimensional slice of this space should be flat.

The function $a(y)$ gives the running string tension

$$T = T_0 a(y), \quad (3.12)$$

where T_0 is the classical string tension (2.14) which we consider as UV data at the UV scale v . It is subject to a renormalization (3.12) within the string theory (3.11).

The condition of the world sheet conformal invariance means the vanishing of β -functions for the 2d theory (3.11). The latter conditions coincide with equations of motion for the 5d effective “gravity” [30]

$$S_{gr} = \frac{1}{2\kappa} \int d^5x \sqrt{g} \exp(-2\Phi) \left[R + 2(D_M \Phi)^2 + 4V(0) + \dots \right]. \quad (3.13)$$

Here $M = 1, \dots, 5$, R is 5d curvature, Φ is the dilaton and $V(0)$ stands for the “cosmological constant” which is the value of the scalar potential at zero. Dots in (3.13) represents other background fields of the string theory (3.11) to be included in (3.13).

Generically (3.13) is relatively useless because we don’t know terms associated with these additional background fields. However, we can look at it as at an effective low energy theory at scales below the string scale \sqrt{T} . Then all string states can be integrated out and we are left with an effective theory of “gravity” for a few light fields. In this setup string is considered as moving in a slow varying classical gravitational background. After finding a solution of gravitational equations of motion we have to check that the curvature of the 5d space is much smaller than the string scale T .

Following [14, 13] we include in (3.13) the $U(1)$ R-R 5-form F besides the 5d metric g_{MN} and the dilaton Φ . With these fields taken into account Einstein equations of motion take the form

$$R_{MN} + 2D_M D_N \Phi + T \exp(2\Phi) \left[F_{MKLPQ} F_N^{KLPQ} \right]$$

$$\left. -\frac{1}{10}g_{MN}F_{SKLPQ}F^{SKLPQ}\right] = 0, \quad (3.14)$$

while the equation balancing the central charge is

$$R - 4(D_M\Phi)^2 + 4D_MD^M\Phi + 4V(0) + \pi T(10 - d) = 0, \quad (3.15)$$

where D_M is the covariant derivative. The last term in (3.15) comes from the anomaly [31]. It is nonzero for the non-critical dimension $d = 5$. Since we started from $\mathcal{N}=2$ QED we assume a target space supersymmetry for the string theory (3.11), thus the critical dimension would correspond to $d = 10$. The equation for the $U(1)$ 5-form reads

$$D^MF_{MKLPQ} = 0 \quad (3.16)$$

The metric in our 5d “gravity” has a special form determined by the single function $a(y)$ (see (3.11))

$$ds^2 = a(y)(dx_\mu)^2 + (dy)^2. \quad (3.17)$$

With this ansatz for the metric eqs. (3.14)-(3.16) were studied in [13]. The solution for (3.16) corresponds to a constant 5-form field strength

$$F_{MKLPQ} = \frac{f}{\sqrt{g}}\epsilon_{MKLPQ} = \frac{f}{a^2}\epsilon_{MKLPQ}, \quad (3.18)$$

where f is a dimensionless constant. Within the large N approach of [14] $f \sim N$, so it is natural to assume that $f \sim 1$ in the case at hand. Substituting this result back into eqs. (3.14), (3.15) we get

$$-\frac{1}{2}\frac{a''}{a} - \frac{1}{2}\frac{a'^2}{a^2} + \frac{a'}{a}\Phi' = Tf^2\exp(2\Phi) \quad (3.19)$$

for $\mu\nu$ components of (3.14) and

$$-2\frac{a''}{a} + \frac{a'^2}{a^2} + 2\Phi'' = Tf^2\exp(2\Phi) \quad (3.20)$$

for the 55 component. The equation for zero central charge becomes ⁵

$$-\frac{1}{2}\Phi'' + \Phi'^2 - \frac{a'}{a}\Phi' - \frac{5}{4}Tf^2\exp(2\Phi) = V(0) + \frac{5\pi}{4}T \quad (3.21)$$

⁵In fact, it is a certain linear combination of the equation (3.15) and eqs. (3.19), (3.20).

Here prime stands for the derivative with respect to y .

These equations admit [13] the solution with AdS metric

$$a(y) = \exp\left(2\frac{y}{r_0}\right) \quad (3.22)$$

and constant dilaton

$$\Phi = \Phi_0. \quad (3.23)$$

Here r_0 is the radius of the AdS space. We can trust our “gravity” solution if this radius is large enough in string units [14],

$$r_0 \gg 1/\sqrt{T}. \quad (3.24)$$

Substituting (3.22) into eqs. (3.19)-(3.21) we find

$$f^2 \exp(2\Phi_0) = -\frac{4}{T r_0^2} \quad (3.25)$$

and

$$r_0^2 = \frac{20}{4V(0) + 5\pi T}. \quad (3.26)$$

From (3.26) we see that in order to fulfill the condition (3.24) we need some cancelation between the “cosmological constant” term $V(0)$ and the contribution due to the anomaly in (3.26). Unfortunately, we don’t know the scalar potential for our 5d “gravity” and cannot check (3.24) on the gravity side.⁶ We will come back to this issue below and use information on the field theory side to show that (3.24) is fulfilled.

Now let us discuss the meaning of the Liouville coordinate y in the 4d field theory. It was suspected for a long time that this coordinate has a meaning of RG scale from the point of view of 4d field theory. In [14] this interpretation was formulated explicitly. To be more specific, it is convenient to introduce a new coordinate r instead of y as

$$r = r_0 \exp \frac{y}{r_0}. \quad (3.27)$$

⁶In principle, certain information about scalar potential can be extracted by the dimensional reduction from 10d supergravity [32]. We do not follow this approach in this paper because our 5d “gravity” is an effective low energy theory which has nothing to do with real 10d gravity.

In terms of this coordinate the AdS metric looks like

$$ds^2 = \frac{r^2}{r_0^2} (dx^\mu)^2 + r_0^2 \frac{(dr)^2}{r^2} . \quad (3.28)$$

Now following [14] we introduce the coordinate

$$u = T r \quad (3.29)$$

which has dimension of energy and identify it with the energy scale of the RG flow, $u = \mu$ (see [33] for the discussion of the normalization in this identification).

In particular, large u , $u \gtrsim u_0$ (here $u_0 = T r_0$) corresponds to UV region in the field theory, while moving to small u towards the throat of the AdS space is associated with the RG flow to the IR. As the mass of the photon ($\sim v$) serves as a UV cutoff for our effective string theory of ANO string, it is natural to identify

$$u_0 = v. \quad (3.30)$$

This identification shows immediately that the AdS radius is large in string units,

$$r_0 = \frac{u_0}{T} = \frac{v}{T} \gg \frac{1}{\sqrt{T}}, \quad (3.31)$$

where we use the condition of smallness of the string tension (3.8). This result shows that we can trust the “gravity” solution.

It was shown in [14, 15, 16] that AdS metric in the 5d gravity corresponds to a conformal invariant field theory in 4d. To understand this note, that the solution (3.22), (3.23) has constant dilaton and zero values of other fields which we do not include in the effective 5d “gravity” (3.13). These fields play the role of “coupling constants” in the 4d field theory [15, 16]. As soon as coupling constants do not run with the RG energy scale u we are dealing with CFT. We obtained our string theory under condition (3.8) as an effective theory at the AD point on the Higgs branch. Therefore the conformal theory in question is the CFT_{AD}^H which we discussed in the previous subsection.

Now let us see what does the “gravity” solution with AdS metric (3.28) gives for the renormalized string tension (3.12). Clearly, 4d conformal invariance means that $T = 0$. This was explicitly shown in [34] by the calculation of the Wilson loop in the AdS_5 background. This result is qualitatively clear from eq. (3.12) with function $a(y)$ given by (3.22). The string goes all the

way down to the throat of the AdS space at $u = 0$ ($y = -\infty$) producing the zero result for the tension.

To get a non-zero string tension we have to prevent the string from penetrating into the throat. This can be done by the conformal symmetry breaking at some scale u_{CB} associated with the kink (domain wall) solution of 5d “gravity” separating AdS region at large u from a different AdS regime in the IR at small u (cf. [32, 35, 36]). We consider this breaking in the next section.

Note, in conclusion of this section that the result $T = 0$ which follows from the AdS solution of the 5d “gravity” coincides with our field theory expectations (3.9) showing selfconsistency of our approach.

4 Deformation of the AdS metric

In general, the 4d CFT can be driven away from criticality by a relevant scalar operator O with conformal dimension $\Delta \leq 4$ by adding the term

$$\int d^4x \sigma_0 O(x) \quad (4.1)$$

to the action. Here σ_0 is a “coupling constant”. From the 5d “gravity” point of view this constant becomes a scalar field $\sigma(r)$ with the boundary value $\sigma = \sigma_0$ at the UV boundary $r = r_0$ [15, 16]. Near the boundary at large r it behaves as

$$\sigma = \sigma_0 \left(\frac{r_0}{r} \right)^{4-\Delta}. \quad (4.2)$$

In the theory at hand we associate the breaking of the conformal invariance with moving slightly away from AD point. Then the monopole mass becomes non-zero, see (3.2). Thus, the relevant deformation in question is the monopole mass term with the conformal dimension $\Delta = 2$. On the “gravity” side we have to include the scalar field σ in our 5d “gravity” (3.13) with the boundary condition

$$\sigma_0 = \frac{m_m^2}{u_0^2}, \quad (4.3)$$

which is determined by the small monopole mass m_m at the scale $u = u_0$.

Einstein equations of motion modify as

$$-\frac{1}{2} \frac{a''}{a} - \frac{1}{2} \frac{a'^2}{a^2} + \frac{a'}{a} \Phi' = T f^2 \exp(2\Phi),$$

$$\begin{aligned}
-2\frac{a''}{a} + \frac{a'^2}{a^2} + 2\Phi'' + 2\sigma'^2 &= T f^2 \exp(2\Phi), \\
-\frac{1}{2}\Phi'' + \Phi'^2 - \frac{a'}{a}\Phi' - \frac{5}{4}T f^2 \exp(2\Phi) &= V(\sigma) + \frac{5\pi}{4}T.
\end{aligned} \tag{4.4}$$

Also we have additional equation for σ

$$-\sigma'' + 2\Phi'\sigma' - 2\frac{a'}{a}\sigma' + \frac{\partial V(\sigma)}{\partial \sigma} = 0. \tag{4.5}$$

Here $V(\sigma)$ is the potential for the scalar σ which we unfortunately don't know.

Below we develop a perturbation theory near the AdS metric at large r to make an estimate of the scale u_{CB} at which the kink solution takes over and destroys the AdS metric. We assume that the scalar potential has the following expansion

$$V(\sigma) = V(0) + \frac{1}{2}m_\sigma^2\sigma^2 + \frac{\lambda}{r_0^2}\sigma^4 + \dots, \tag{4.6}$$

where λ is dimensionless constant and m_σ is the mass of the scalar σ . Eq. (4.5) gives for this mass

$$m_\sigma^2 = -\frac{4}{r_0^2} \tag{4.7}$$

in accordance with the general result [15, 16]

$$\Delta = 2 + \sqrt{4 + m_\sigma^2 r_0^2} \tag{4.8}$$

relating the conformal dimension of operator O in 4d field theory and the mass of the corresponding scalar field in 5d gravity ⁷.

The solution of equations of motion (4.4) to the second order in the perturbation around the AdS metric looks like

$$\begin{aligned}
a(y) &= e^{2\frac{y}{r_0}} \left[1 + \left(\frac{\lambda}{8} - \frac{1}{25} \right) \sigma_0^4 e^{-8\frac{y}{r_0}} + \dots \right], \\
\Phi &= \Phi_0 - \frac{\sigma_0^2}{5} e^{-4\frac{y}{r_0}} + \frac{\sigma_0^4}{25} e^{-8\frac{y}{r_0}} + \dots, \\
\sigma &= \sigma_0 e^{-2\frac{y}{r_0}} + \left(\frac{\lambda}{4} - \frac{1}{5} \right) \sigma_0^3 e^{-6\frac{y}{r_0}} + \dots.
\end{aligned} \tag{4.9}$$

⁷ Note that the value of mass in (4.7) is at the border of stability, see [37].

Here the metric, the dilaton and the scalar σ expressed in terms of the boundary value σ_0 which is determined by the small monopole mass via the boundary condition (4.3). The dilaton expectation value Φ_0 is constrained by eq. (3.25). To find higher terms in this expansion (or exact solution for the kink) we need to know the form of the potential (4.6).

Note, that generally speaking the equation (4.5) admits two solutions with behavior r_0^2/r^2 and $\log(r_0^2/r^2)r_0^2/r^2$ at large r . One can show, however, that the second solution does not lead to a consistent solution of other equations of motion in (4.4).

For generic couplings λ ($\lambda \sim 1$) corrections in (4.9) becomes of order of one and destroy the AdS metric at y_{CB} determined by

$$\exp\left(2\frac{y_{CB}}{r_0}\right) \sim \sigma_0. \quad (4.10)$$

In terms of the RG scale variable u the above equation reads

$$\frac{u_{CB}^2}{u_0^2} \sim \sigma_0. \quad (4.11)$$

Substituting here the boundary value of σ_0 (4.3) we get the scale of the conformal symmetry breaking

$$\frac{u_{CB}^2}{u_0^2} \sim \frac{m_m^2}{v^2}. \quad (4.12)$$

Now let us make an estimate of the value of the renormalized string tension at the scale u_{CB} . Eq. (3.12) gives

$$T = T_0 \frac{u^2}{u_0^2}, \quad (4.13)$$

where we use $a(u) = u^2/u_0^2$ for the AdS metric. Assuming that the kink stops the string from penetrating into the throat of the AdS space and determines the scale u_{CB} to be substituted into eq. (4.13) we get

$$T \sim \frac{m_m^2}{\log vL}, \quad (4.14)$$

where we use (4.12) and the expression (2.14) for the bare string tension at the UV scale v . Recall that the length of the string L appears here because

we do not have infinitely long flux tubes on the Higgs branch where quarks are strictly massless, see subsection 2.2.

The estimate (4.14) is our final result for the string tension near the AD point. To prove it rigorously (and to work out the coefficient in (4.14)) one has to find the exact solution for the kink and to calculate the Wilson loop in the kink background (cf. [34, 35, 36]). This is left for a future work.

Note, that the result (4.14) satisfy conditions (3.8) and (3.9) showing the consistency of the 5d “gravity” description with field theory expectations. In particular, (4.14) gives zero tension at the AD point. As we already explained, the string becomes tensionless, however still remains to be a stringy object because its transverse size $\sim v^{-1}$ is finite. This is an interesting example of the non-trivial conformal theory CFT_{AD}^H containing massless quarks and tensionless ANO strings with massless monopoles attached to ends of these strings.

Now let us estimate the value of the string coupling constant and show that it is small, $g_s \ll 1$. First note, that as we already mentioned in section 3 the radius of the AdS space is large in string units. Substituting (4.14) into (3.31) we get

$$r_0 \sim \frac{v}{m_m^2}. \quad (4.15)$$

where we drop the logarithm factor. Now eq. (3.25) gives for the closed string coupling constant ($g_s^2 = \exp 2\Phi_0$)

$$g_s^2 f^2 \sim \frac{m_m^2}{v^2} \ll 1. \quad (4.16)$$

As we already mention we assume that the field strength of the RR 5-form is of order one, $f \sim 1$. Then (4.16) gives $g_s \ll 1$.

We can also estimate the string coupling from the field theory side. The open string coupling constant measures the probability that the string is broken by the monopole-anti-monopole pair production. This probability is of order of [38]

$$g_s \sim \exp\left(-c \frac{m_m^2}{T}\right), \quad (4.17)$$

where the positive constant c can be, in principle calculated. Taking into account the logarithm factor in (4.14) we get

$$g_s \sim \left(\frac{m_m}{v}\right)^\gamma \ll 1, \quad (4.18)$$

where $\gamma > 0$. Here we use that the typical length of the string is of order of $L \sim 1/\sqrt{T}$ for small hadron spins. The reason why the string coupling constant turns out to be small is that monopoles, although light with respect to photon, appear to be heavy with respect to the string scale \sqrt{T} because of the logarithm factor in (4.14). Therefore, the monopole-anti-monopole production is suppressed.

Although we don't know γ in the field theory expression (4.18) as well as we don't know f^2 on the “gravity” side, the two estimates (4.18) and (4.16) are consistent with each other and show that our string theory is in the weak coupling.

5 Discussion

In this paper we considered ANO flux tubes on the Higgs branch near the AD point in $\mathcal{N}=2$ QCD. The effective low energy QED describing the theory near the root of the Higgs branch becomes strongly coupled when we approach the AD point. Thus, the semi-classical analysis is no longer valid. We present arguments based on the consideration of instanton induced higher derivative corrections on the Higgs branch that the ANO string tension is small, much smaller than the scale determined by the quark condensate, see (3.8), (3.9). This condition ensures that the ANO flux tube is long and thin and can be described by the non-critical string theory with the world sheet conformal invariance. This leads us to the string moving in 5d space with curved fifth (Liouville) coordinate [12, 13]. At the AD point the 4d conformal invariance on the field theory side corresponds to AdS background metric of the 5d space. We also considered the breaking of the conformal invariance by moving slightly away from the AD point. From 5d “gravity” equations of motion we found that the renormalized ANO string tension is determined by the small mass of monopoles, see (4.14).

The main lesson to learn is that once we make the confining matter light the ANO string becomes light too and does not “want” to live in four dimensions. It goes into a five dimensional space. This happens already at the hadron scale.

Note, that we do not use the logic of large N in this paper [14]. The radius of the AdS space remains unfixed within the 5d “gravity” approach, see (3.26). Instead, we use the field theory arguments to show that it is large in string units, see (4.15). Thus the “gravity” solution can be trusted.

At first sight it seems surprising that 5d “gravity” gives reasonable results. It seems to “know” almost nothing about the field theory at the AD point under consideration. Still the results we obtained from 5d “gravity” are consistent with the field theory expectations. This means that the 5d “gravity” description is quite universal. The only information from the field theory side that we use formulating the string theory (3.11) and its effective 5d “gravity” description is the existence of two scales v and m_m , subject to the condition $m_m \ll v$ as well as relations (3.8), (3.9) for the string tension. On the 5d “gravity” side we identified the UV scale v with the radius of the AdS space u_0 (see (3.30)) and the small monopole mass m_m with the boundary value of the 5d scalar σ , see (4.3). Then Einstein equations give the estimate (4.12) which tells us that the scale of the conformal symmetry breaking is given by m_m , which is quite obvious conclusion from the field theory point of view. Then the result (4.14) comes from the expression for the running string tension (4.13) in the AdS background [12, 13].

Of course, to find the exact solution for the kink (domain wall) we need to use much more information about our field theory near AD point. First, we have to impose $\mathcal{N}=2$ supersymmetry. On the 5d “gravity” side this means imposing $\mathcal{N} = 4$ supersymmetry for the AdS background which corresponds to CFT_{AD}^H at large u (cf. [32]). The kink should be 1/2-BPS solution preserving the $\mathcal{N}=2$ supersymmetry. It should satisfy first order differential equations. This kink separates the UV AdS region associated with CFT_{AD}^H from the different AdS regime in the IR. The latter should correspond to the free theory of massless quark moduli on the Higgs branch which emerges at scales well below the string scale \sqrt{T} . We also have to impose global $SU(2)_R$ symmetry which becomes a gauge symmetry in 5d “gravity”. Hopefully this information would be enough to fix the scalar potential in 5d “supergravity” and find the kink solution.

Another important open problem is related to the presence of massless quarks on the Higgs branch. The number of massless quark moduli specify precisely the CFT_{AD}^H we are dealing with. In particular, it determines fractional conformal dimensions of various operators. The presence of massless quark moduli should be taken into account in the 5d “gravity” description. In ref. [39] fractional dimensions were related to the spectrum of Laplacian operator in the geometry determined by 3-branes moving near 7-branes singularities. It would be interesting to find a 5d analog of this description.

Acknowledgments

The author is grateful to Alexandr Gorsky, Gennady Danilov, Andrei Marshakov, Erich Poppitz, Mikhail Shifman, Arkady Vainshtein and, in particular, to Alexei Morozov for helpful discussions. The author also would like to thank the Theoretical Physics Institute at the University of Minnesota for hospitality and support. This work is also supported by Russian Foundation for Basic Research under grant No. 99-02-16576 and by the US Civilian Research and Development Foundation under grant No. RP1-2108.

References

- [1] N. Seiberg and E. Witten, hep-th/9407087; Nucl.Phys.B **426** (1994) 19.
- [2] N. Seiberg and E. Witten, hep-th/9408099; Nucl.Phys.B **431** (1994) 484.
- [3] S. Mandelstam, Phys.Rep. **23C** (1976) 145;
G.'t Hooft, in Proceed.of the Europ.Phys.Soc. 1975, ed.by A.Zichichi (Editrice Compositori, Bologna, 1976), p.1225.
- [4] A.A. Abrikosov, Sov.Phys.JETP, **32** (1957) 1442;
H.B. Nielsen and P. Olesen, Nucl.Phys. **B61** (1973) 45.
- [5] A. Hanany, M. Strassler, and A. Zaffaroni, hep-th/9707244; Nucl.Phys.B **513** (1998) 87.
- [6] A. Yung, hep-th/9906243; Nucl.Phys. **B562** (1999) 191.
- [7] A. Yung, hep-th/0005088; Proceedings of the XXXIV PNPI Winter School, Repino, St. Petersburg, 2000.
- [8] A. Bilal and F. Ferrari, hep-th/9706145; Nucl.Phys.B **516** (1998) 175.
- [9] P.C. Argyres and M.R. Douglas, hep-th/9505062; Nucl.Phys.B **448** (1995) 93.
- [10] P.C. Argyres, M.R. Plesser, N. Seiberg, and E. Witten, hep-th/9511154; Nucl.Phys.B **461** (1996) 71.
- [11] T. Eguchi, K. Hori, K. Ito, and S. Yang, hep-th/9603002; Nucl.Phys.B **471** (1996) 430.

- [12] A. Polyakov, hep-th/9711002; Nucl.Phys. Proc. Suppl. **68** (1998) 1.
- [13] A. Polyakov, hep-th/9809057; Int. J. Mod. Phys. A **14** (1999) 645.
- [14] J. Maldacena, hep-th/9711200; Adv. Theor. Math. Phys. **2** (1998) 231.
- [15] S. S.Gubser, I. R. Klebanov and A. M. Polyakov, hep-th/9802109; Phys. Lett. B **428** (1998) 105.
- [16] E. Witten, hep-th/9802150; Adv. Theor. Math. Phys. **2** (1998) 253.
- [17] A. Vainshtein and A. Yung, *Type I superconductivity upon monopole condensation in Seiberg-Witten theory*, hep-th/0012250;
- [18] P. Argyres, M. Plesser and N. Seiberg, hep-th/9603042; Nucl. Phys. B **471** (1996) 159.
- [19] A. Yung, hep-th/9705181; Nucl. Phys. B **512** (1998) 79.
- [20] A. Yung, hep-th/9605096; Nucl. Phys. B **485** (1997) 38.
- [21] A. Penin, V. Rubakov, P. Tinyakov and S. Troitsky, hep-th/9609257; Phys.Lett B **389** (1996) 13.
- [22] A. Achucarro, M. de Roo and L. Huiszoon, Phys.Lett. B **424** (1998) 288.
- [23] K. Intriligator and N. Seiberg, hep-th/9509066; Nucl. Phys. BC (Proc. Suppl.) **45** (1996) 1.
- [24] A. Gorsky, A. Vainshtein and A. Yung, hep-th/0004087; Nucl. Phys. B **584** (2000) 197.
- [25] P. Orland, hep-th/9404140; Nucl.Phys. B **428** (1994) 221;
M. Sato and S. Yahikozawa, hep-th/9406208; Nucl.Phys.B **436** (1995) 100.
- [26] M. Baker and R. Steinke, hep-ph/9905375; Phys. Lett. B **474** (2000) 67
- [27] J. Polchinski and A. Strominger, Phys.Rev.Lett. **67** (1991) 1681.
- [28] E. Akhmedov, M. Chernodub, M. Polikarpov, and M. Zubkov, Phys.Rev.D **53** (1996) 2087.

- [29] A. Polyakov, Nucl.Phys.B **268** (1986) 406.
- [30] C. Callan, E. Martinec, M. Perry and D. Friedan, Nucl.Phys.B **262** (1985) 593.
- [31] A. Polyakov, Phys. Lett. B **103** (1981) 207.
- [32] D. Z. Freedman, S. S. Gubser, K. Pilch and N. P. Warner, *Renormalization group flows from holography- supersymmetry and a c-theorem*, hep-th/9904017.
- [33] A. Peet and J. Polchinski, hep-th/9809022; Phys. Rev. D **59** (1999) 065011.
- [34] J. Maldacena, hep-th/9803002; Phys. Rev. Lett. **80** (1998) 4859.
- [35] L. Girardello, M. Petrini, M. Poratti and A. Zaffaroni, hep-th/9903026; JHEP **9905** (1999) 026.
- [36] L. Girardello, M. Petrini, M. Poratti and A. Zaffaroni, hep-th/9909047; Nucl.Phys.B **569** (2000) 451.
- [37] P. Breitenlohner and D. Z. Freedman, Phys. Lett. B **115** (1982) 197; Ann. Phys. **144** (1982) 197.
- [38] I. Novikov, M. Shifman and A. Yung, to appear.
- [39] O. Aharony, A. Fayyazuddin and J. Maldacena, hep-th/9806159; JHEP **9807** (1998) 013.